CEP Calculations for a Rocket with Different Control Systems

Gregor Gregoriou*
ENVIRO GmbH, Munich, Federal Republic of Germany

Extensive aeromechanical calculations were performed for a ground-to-ground rocket with a range of 120 km. The first objective was to investigate the influence of the control system on the circular error probability (CEP) with respect to the assumed error budget. Results are presented for three different cases, namely: no control, attitude control, and control with the aid of a strapdown inertial platform. It is demonstrated that only the application of the latter permits compensating for the effect of even large total-impulse deviations. Rocket control is carried out by fin deflections only. Examples are also given that show the influence of static margin and axial-force coefficients on the CEP. The second objective was to find out the maximum total-impulse deviation that could be completely neutralized by means of aerodynamic controls.

Nomenclature

A	= acceleration
CEP	= circular error probability (in % of total range)
C_l, C_n, C_m	= aerodynamic coefficients for rolling, yawing, and pitching moments
$C_{l_p}, C_{n_r}, C_{m_q}$	= damping derivatives for rolling, yawing, and pitching
C_X, C_Y, C_Z	= aerodynamic coefficients for axial, side, and normal forces
D	= reference length
f	= damping factor of actuator or measuring device
g	= Earth acceleration
I_x, I_y, I_z	= moments of inertia (body-fixed axes)
K	= gain of actuator or measuring device
m	= rocket mass
p,q,r	= angular rates
$Q1,\ldots,Q4$	= quaternions
S	= reference area
T	= transformation matrix
TI	= total impulse, = 1 denotes nominal value
T_l, T_n, T_m	= moment components due to thrust (body-fixed axes)
T_x, T_y, T_z	= force components due to thrust (body-fixed axes)
t	= time
u,v,w	= body-fixed velocity components
V	= resultant velocity
x,y,z	= body-fixed axes
α	= angle of attack
β	= angle of sideslip
γ,χ	= velocity vector angles
η,ζ,δ	= control deflections for pitching, yawing, and rolling
θ,ϕ,ψ	= Euler angles
ρ	= air density
ω	= frequency of actuator or measuring device
Indices	
M	= measured
C	= commanded
AC	= acceleration
R	= reference

For the explanation of symbols see also Figs. 1 and 2.

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*President. Member AIAA.

Introduction

THE effect of different errors on the dispersion of unguided rockets has been investigated in detail in the past.[1-3] In the case of a ballistic ground-to-ground rocket with a range of 120 km, the expected deviations from the nominal impact point due to anticipated errors are very large for practical applications. Therefore, it is necessary to equip the rocket with an appropriate control system; one that permits suppressing the said deviations to an acceptable level. An extensive investigation was performed within this framework for a specific rocket design using a six degree-of-freedom (d.o.f.) trajectory-computing code. The final results of the investigation are presented in the form of circular-error-probability (CEP) values.

The total task was subdivided into the following steps: 1) establishing the database for aerodynamics; 2) establishing the error budget; and 3) performing trajectory calculations and defining the CEP for a nonrotating rocket (without control), a rotating rocket (without control), a rotating and a nonrotating rocket with attitude control, and a nonrotating rocket with inertial platform.

Rocket motion was controlled in the latter two cases by deflecting the fins only. Further, the effect of varying the aerodynamic coefficients C_Z and C_X, and the static margin on the CEP, were also investigated.

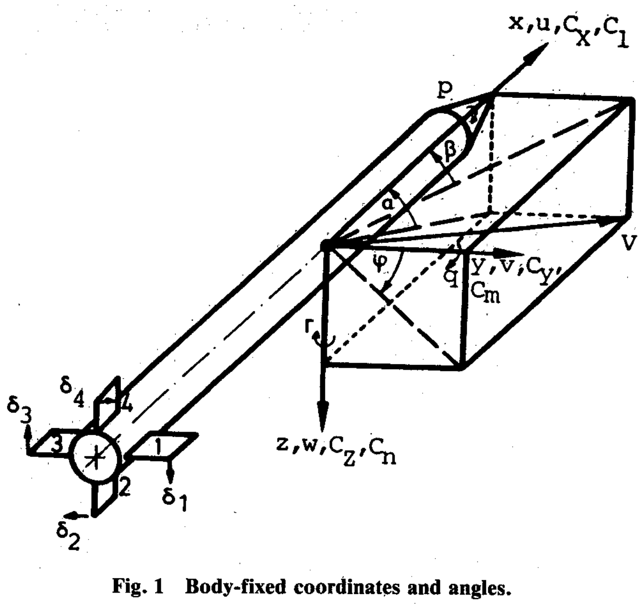

Fig. 1 Body-fixed coordinates and angles.

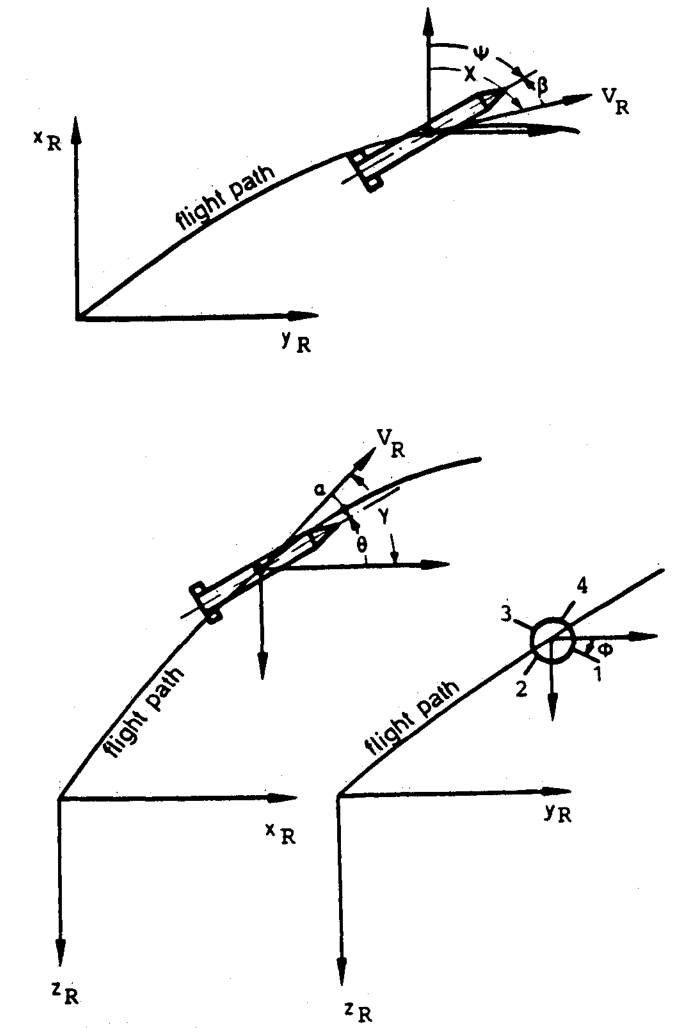

Fig. 2 Euler angles θ, ψ, and ϕ, and velocity vector orientation angles γ and χ.

The aerodynamic coefficients were first calculated[4] and then measured[5] with the aid of wind-tunnel tests.

The assumed error budget comprises the aiming error, total-impulse deviation, thrust and aerodynamic misalignment, platform drift, mal-launch, and wind disturbances. The latter were assumed to amount to 1 m/s constant wind velocity distributed uniformly. Experience has shown that, in many cases, this assumption suffices for preliminary design purposes, if wind measurements and, as their result, launcher corrections are to be undertaken. Consideration of different wind profiles is very complex and should be the task of a separate study. In the case of controlled rocket flight, a platform drift of 1 deg/h was considered as well as a resolution of 0.03 deg with respect to the fin deflections. Other error sources were omitted. It was expected that their contribution will not significantly change the findings of this study. Its main purpose was not to define precisely the lowest achievable CEP but to compare the results of different control systems assuming rather high values for the most important errors. Further, the highest value of total-impulse deviation that could be completely neutralized by means of aerodynamic controls was of interest. No published report exists in this field to the author's knowledge.

All trajectory calculations were performed for the standard atmosphere and at an elevation angle of 62.5 deg, which corresponds to the maximum range. The applied 6 d.o.f. trajectory-computing code considers the influence of Earth rotation and curvature. The CEP calculation refers to the impact point deviations of the rocket due to the assumed errors and is carried out by applying statistical methods.

Mathematical Formulation

The motion of a rocket in space with 6 d.o.f. is described with the aid of six differential equations for the forces and moments.[6] Consideration of the kinematic equations and Euler angles (the latter replaced by four quaternions[7,8]) finally leads to a system of 13 differential equations, which can be written as

Equations of force:

$$\dot{u} = (C_X \frac{\rho}{2} V^2 S + T_x)/m + rv - qw + gQT1 \qquad (1)$$

$$\dot{v} = (C_Y \frac{\rho}{2} V^2 S + T_y)/m + pw - ru + gQT2 \qquad (2)$$

$$\dot{w} = (C_Z \frac{\rho}{2} V^2 S + T_z)/m + qu - pv + gQT3 \qquad (3)$$

Equations of moment:

$$\dot{p} = \left\{\left[(C_l V + C_{l_p} Dp) \frac{\rho}{2} VDS + T_l\right] + (I_y - I_z)\, qr\right\}/I_x \qquad (4)$$

$$\dot{q} = \left\{\left[(C_m V + C_{m_q} Dq) \frac{\rho}{2} VDS + T_m\right] + (I_z - I_x)\, pr\right\}/I_y \qquad (5)$$

$$\dot{r} = \left\{\left[(C_n V + C_{n_r} Dr) \frac{\rho}{2} VDS + T_n\right] + (I_x - I_y)\, pq\right\}/I_z \qquad (6)$$

Kinematic equations:

$$\dot{x} = uQT4 + vQT5 + wQT6 \qquad (7)$$

$$\dot{y} = uQT7 + vQT8 + wQT9 \qquad (8)$$

$$\dot{z} = uQT1 + vQT2 + wQT3 \qquad (9)$$

Quaternions (Euler angles):

$$\dot{Q}1 = 0.5(Q2p + Q3q + Q4r) \qquad (10)$$

$$\dot{Q}2 = 0.5(-Q1p - Q4q + Q3r) \qquad (11)$$

$$\dot{Q}3 = 0.5(Q4p - Q1q - Q2r) \qquad (12)$$

$$\dot{Q}4 = 0.5(-Q3p + Q2q - Q1r) \qquad (13)$$

$QT1 \ldots QT9$ are functions of the four quaternions $Q1 \ldots Q4$. The preceding kinematic equations normally refer to the flat Earth geodetic coordinate system located in the launcher. Additional consideration of the Earth's rotation and curvature can be performed best by transforming the kinematic equations into a coordinate system originating in the Earth's center.

The integration of the preceding equations with the aid of an appropriate procedure (fourth-order Runge-Kutta, for example) yields the body-fixed velocity components and angular rates, the rocket coordinates with respect to the reference system, and the quaternions at an arbitrary time t. The angles that describe the inclination of the body axes (θ,ϕ,ψ) and velocity vector orientation (γ,χ) as well as the angle of attack and sideslip (α and β) can be expressed by means of the now-known velocities and quaternions. In general, a system of 13 differential equations suffices for the description of all relevant quantities with regard to a ballistic rocket. In the case of a guided rocket, additional differential equations are needed to consider the actuator unit. Assuming a second-order behavior of the latter, we can now formulate the following equation set.[9]

Actuator Unit

$$\ddot{\eta} = \omega^2\left(\eta_C - \frac{2f}{\omega}\dot{\eta} - \eta\right) \qquad (14)$$

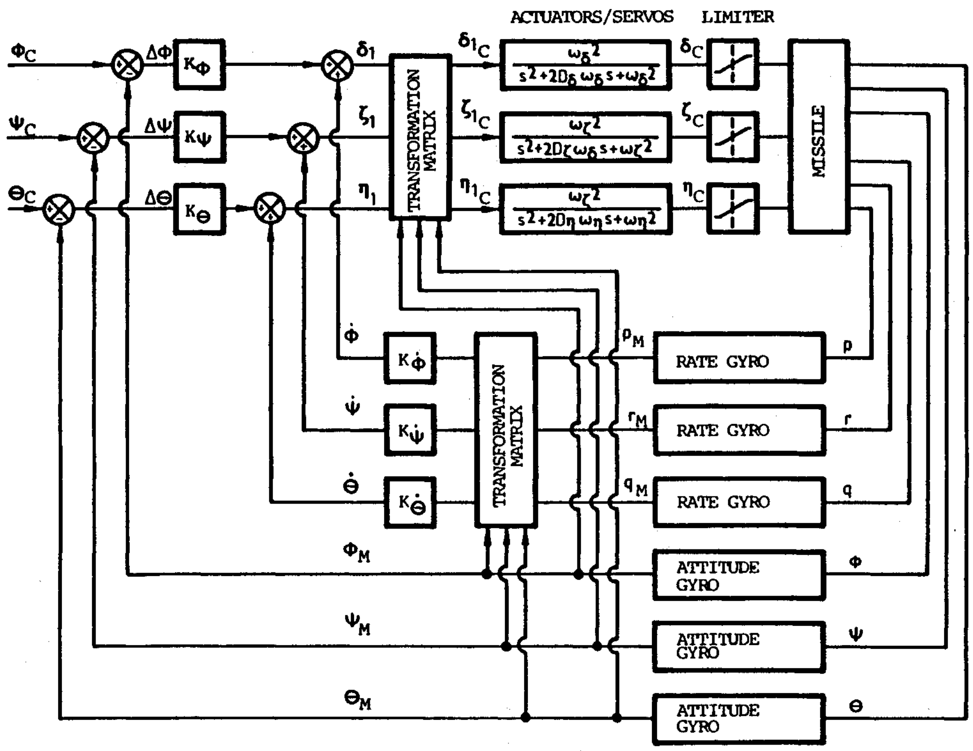

Fig. 3 Block diagram of the attitude control system.

$$\ddot{\zeta} = \omega^2\left(\zeta_C - \frac{2f}{\omega}\dot{\zeta} - \zeta\right) \tag{15}$$

$$\ddot{\delta} = \omega^2\left(\delta_C - \frac{2f}{\omega}\dot{\delta} - \delta\right) \tag{16}$$

The commanded control deflections for pitch, yaw, and roll (η_C, ζ_C, δ_C) are defined as follows:

$$\eta_C = K_\theta(\gamma - \gamma^*) + K_{\dot\theta}\,\dot\theta_M + K_{AC}(A_{z_C} - A_{z_M}) \tag{17}$$

$$\zeta_C = K_\theta(\chi - \chi^*) + K_{\dot\theta}\,\dot\psi_M + K_{AC}(A_{y_C} - A_{y_M}) \tag{18}$$

$$\delta_C = K_\phi(\phi_M - \phi^*) + K_{\dot\phi}\,\dot\phi \tag{19}$$

The asterisk denotes prescribed values (obtained from the nominal trajectory). It has been assumed here that the control system attempts to eliminate the differences between measured and prescribed values of the velocity vector angles and of the accelerations in the y and z directions.

Measuring Devices

The proposed strapdown inertial platform is equipped with rate gyros and accelerometers. Their behavior can also be described by means of differential equations. The following set of equations has been established to predict all of the quantities related to the measuring devices.

Equations for rate gyros:

$$\dddot{\theta} = \omega^2\left(q - \frac{2f}{\omega}\ddot\theta_M - \dot\theta_M\right) \tag{20}$$

$$\dddot{\psi} = \omega^2\left(r - \frac{2f}{\omega}\ddot\psi_M - \dot\psi_M\right) \tag{21}$$

$$\dddot{\phi} = \omega^2\left(p - \frac{2f}{\omega}\ddot\phi_M - \dot\phi_M\right) \tag{22}$$

Equations for accelerometers:

$$\dddot{u} = \omega^2\left(\dot u - \frac{2f}{\omega}\ddot u_M - \dot u_M\right) \tag{23}$$

$$\dddot{v} = \omega^2\left(\dot v - \frac{2f}{\omega}\ddot v_M - \dot v_M\right) \tag{24}$$

$$\dddot{w} = \omega^2\left(\dot w - \frac{2f}{\omega}\ddot w_M - \dot w_M\right) \tag{25}$$

The preceding equation set is valid for the most complex case investigated, namely the design of a control system with an inertial platform. A similar equation set was established for the control system with an attitude platform.

Control System Presentation

The attitude control system used is represented by the block diagram shown in Fig. 3. It is equipped with rate and attitude gyros, which measure the body-fixed angular velocities and attitude angles of the rocket. The controller presented in Fig. 4 was designed for the horizontal, and, accordingly, for the vertical plane, for the control system with an inertial platform. The similarly designed roll controller is not shown here for the reason of brevity. Different controller concepts were investigated with varying success. It turned out that the consideration of accelerations was necessary to successfully compensate for the effect of the expected errors. Roll-stabilized rocket flight was assumed in this case.

The gains for the designed controllers were obtained as functions of the rocket velocity by applying a method similar to that described in Ref. 10. Control system efficiency and stability as well as an adequate dynamic behavior were the main criteria used for gain selection. The influence of the static margin has proved to be very small, at least for the static margin variations considered here. Therefore, the gains were assumed to be independent of the stability. Further, as mentioned earlier, the assumption was made that it suffices for the objectives of the present study to take into account a platform drift of 1 deg/h, a resolution of the actuator unit of 0.03 deg, and to omit less important errors of the measuring devices. A less favorable resolution would have a detrimental influence on the control system efficiency.

The control schemes proposed here are typical, simple, and adequate to the task.[11] However, if CEP minimization were to be achieved, a more complex control scheme would be designed to include additional terms in the equations for

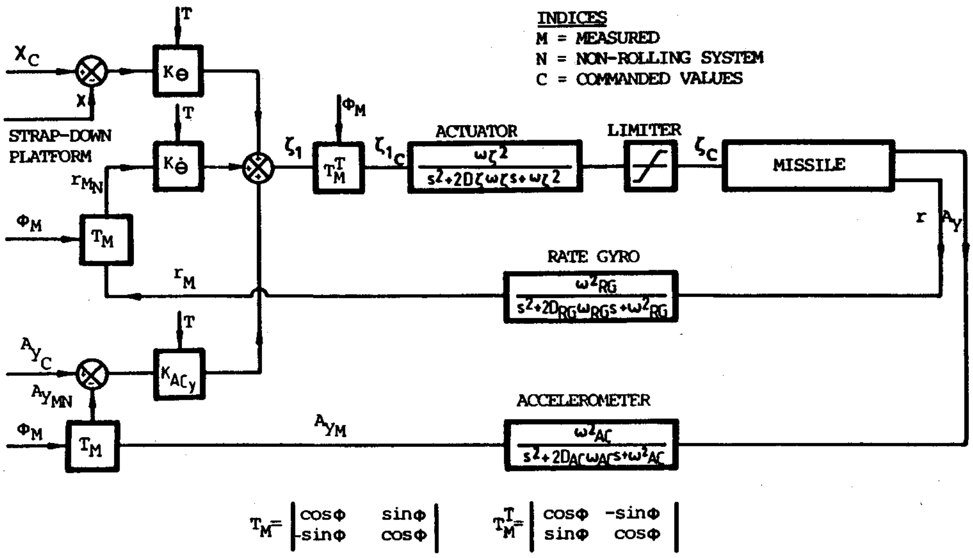

Fig. 4 Block diagram of the horizontal controller (inertial platform).

calculating the commanded control deflections. Such terms might consider the deviation between actual and desired rocket position, respectively, between actual and desired axial velocities.

Brief Discussion of Results Obtained

Nonrotating Rocket (without Control)

A CEP of more than 25% was obtained here. This case is of no practical importance and was investigated only for the sake of completeness. Dominant errors were thrust and aerodynamic misalignment as well as total-impulse deviations.

Rotating Rocket (without Control)

It is well known that rocket spinning is an efficient method for compensating for the effect of the aerodynamic and thrust misalignment on the impact point deviation. Other errors are not suppressed significantly by rocket spinning. Spin rates had to be less than 3 Hz in the present case to avoid occurrence of pitch-roll resonance. However, the selected spin rates are high enough to compensate for said errors. An example of calculated trajectories is shown in Fig. 5. The total-impulse error has a tremendous influence on the range. This is clearly expressed by the CEP values presented in Fig. 6 (curve 1). An optimization procedure, including delayed fin unfolding and variation of the rotation frequency, could lead to slightly lower values. It is felt that the achievable CEP improvement might amount to 10–20%.

Rotating and Nonrotating Rocket with Attitude Control

Rocket dispersion can be reduced by utilizing an attitude platform that measures the actual angular rates and attitude angles for elevation, azimuth, and bank. The control unit then compares the actual with the prescribed attitude angles (obtained from the nominal trajectory calculations). In the event that the actual and prescribed angles differ from one another, the actuator deflects the fins, continuously attempting to eliminate these differences. However, attitude control is not very effective with regard to the influences of the total-impulse deviation, thrust, and aerodynamic misalignment. The two last errors can usually be suppressed more successfully by rotating the rocket.

It turned out that the CEP values obtained with this attitude control version are even slightly higher than with the rotating rocket, as is shown in Fig. 6 (curve 2). Combining rocket rotation (approximately 2 Hz) with attitude control (roll controller eliminated) might lead to far more favorable results (curve 4). The gains of the control system are not optimized. This can improve the presented results to some extent.

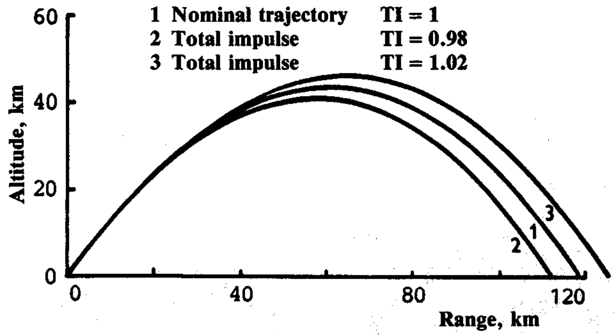

Fig. 5 Range diagram for ballistic flight (rotating rocket).

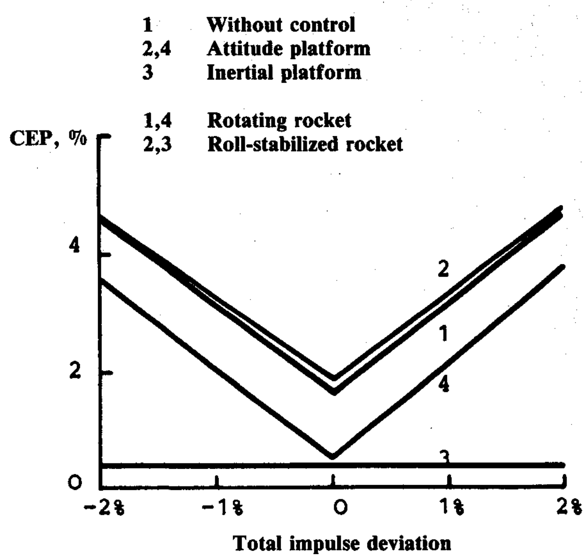

Fig. 6 Influence of total-impulse deviation on CEP.

Nonrotating Rocket with Inertial Platform

A further reduction of the CEP can be achieved by designing a more precise control system that utilizes a strapdown inertial platform equipped with measuring devices for the angular rates and the acceleration components of the rocket. The onboard control unit then compares the actual angles for the velocity vector and the acceleration components with the prescribed values for these quantities (obtained from the

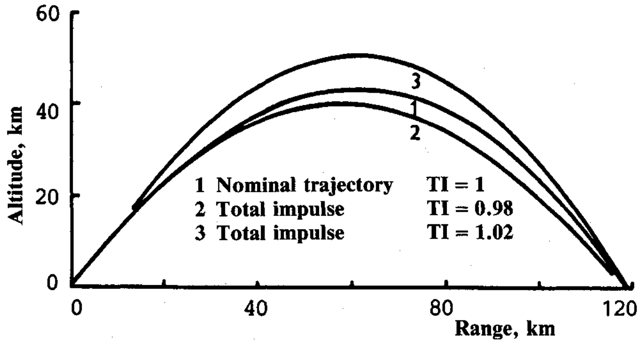

Fig. 7 Range diagram for controlled flight (inertial platform, roll-stabilized rocket).

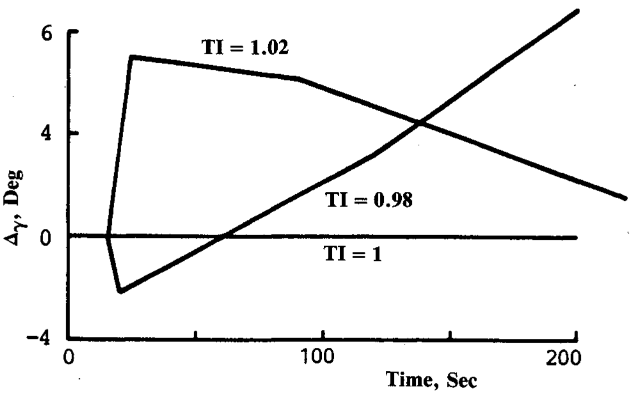

Fig. 8 Additional γ distribution necessary to compensate for the total-impulse error.

nominal trajectory). In the event that actual and prescribed values differ from one another, the actuator unit deflects the fins, attempting to eliminate these differences. Utilization of an inertial platform enables the rocket to alleviate the influence of the total-impulse deviation, as is demonstrated in Figs. 6 and 7, curve 3 (provided that no thrust-terminating mechanism exists). This is performed here by adding a function $\Delta\gamma$ to the prescribed γ^* distribution. An example of the $\Delta\gamma$ characteristic is shown in Fig. 8, where the total-impulse deviation amounts to $\pm 2\%$. $\Delta\gamma$ was defined empirically by trial and error. Although this task seemed to be time-consuming initially, it soon turned out that the user very quickly learns in dialog with the computer how to tailor $\Delta\gamma$ to obtain the desired result. Negligible residual deviations between actual and nominal impact points, load factors below the structural limits, and control deflections less than 15 deg were the requirements that $\Delta\gamma$ had to fulfill.

The total-impulse deviation cannot be measured directly, but only indirectly, best of all by comparing the actual with the nominal burn-out velocity. The latter is defined by integrating the measured longitudinal accelerations. An algorithm that correlates burn-out velocity and total-impulse deviation can easily be obtained with the aid of trajectory calculations.

It is inherent in the designed control system that disturbances acting in a cross-flow plane can be suppressed better than disturbances acting in the longitudinal axis, such as head wind, axial-force differences, or total-impulse deviations. Then, the control system can only cope with longitudinal disturbances by forcing the rocket to perform maneuvers in a cross-flow plane, and this is less effective. The possibility of inserting an additional term in Eq. (17) to consider the difference between actual and nominal axial velocity was not investigated in the present study. Comparison of axial velocities might be the best way to alleviate the effect of axial disturbances. High accuracy of the accelerometer would be required in this case since velocity changes may be very small, depending on the disturbance. However, small velocity changes cause large range deviations.

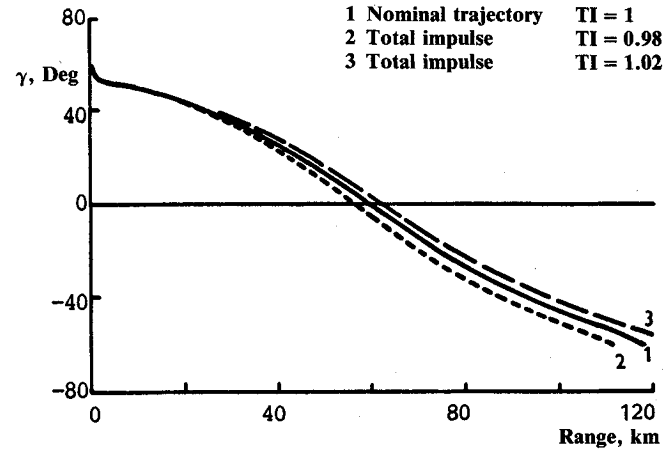

Fig. 9 Characteristic of velocity vector angle γ for ballistic flight (rotating rocket).

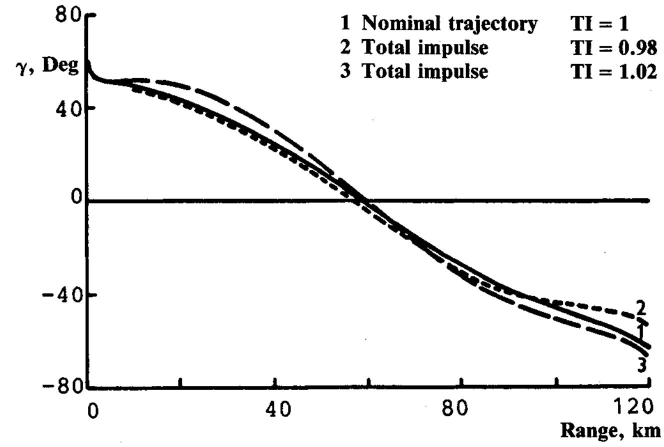

Fig. 10 Characteristic of velocity vector angle γ for controlled flight (inertial platform, roll-stabilized rocket).

A comparison of the γ characteristics for different total-impulse deviations is shown in Fig. 9 for the ballistically flying rocket, and in Fig. 10 for the rocket with an inertial platform. The control system considerably modifies the prescribed γ^* characteristic when attempting to eliminate the expected range deviation due to this error. In the investigated rocket design, the control system can cope with total-impulse deviations of approximately $\pm 2.0\%$. The total CEP obtained with the suggested inertial platform and control system amounts to approximately 0.4%. It is mainly caused by the aerodynamic and thrust misalignment, aiming error, and platform drift. A CEP improvement could be achieved by optimizing the gains.

The demand for fin control deflections is relatively high, as Fig. 11 demonstrates. However, the high angles occur at low-air-density regions so that the expected maximum loads do not exceed the structural limits imposed. The demand for control deflections can be diminished considerably by decreasing the static margin.

Influence of Aerodynamic Coefficients and Static Margin on CEP

A section of the present investigation was devoted to the sensitivity of the CEP with respect to the variation of the normal- and axial-force coefficients, and static margin. As was expected, the normal-force coefficient does not significantly affect the CEP, at least for the variations investigated within the range of -3 to $+3\%$.

The influence of the axial-force coefficient is more pronounced, depending on the control system applied. Some typical results are presented in Fig. 12. Varying the axial-force coefficient for 1% or -1% will not strongly affect the CEP of the rocket, except in the case of the control system with the inertial platform. A CEP increase of 50% is estimated for the

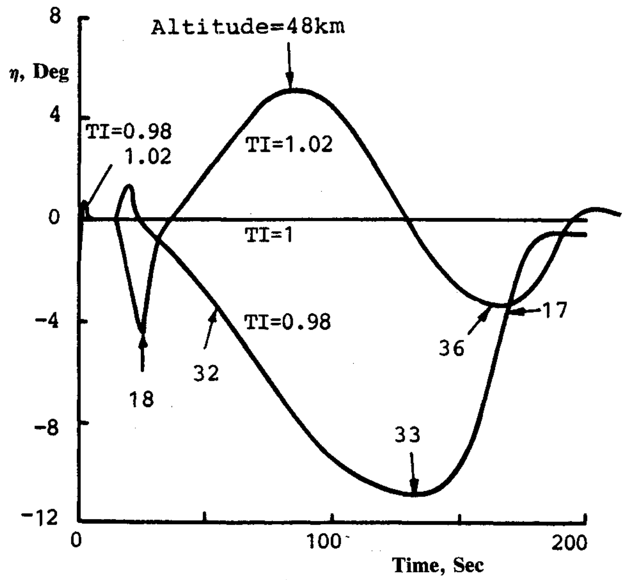

Fig. 11 Pitch-control deflection angle vs time (inertial platform, roll-stabilized rocket).

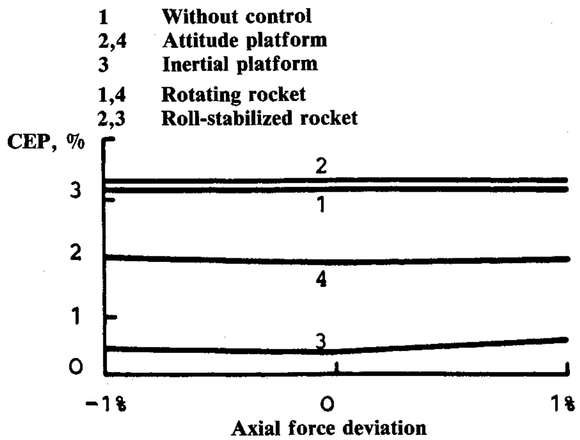

Fig. 12 Influence of axial-force deviation on CEP.

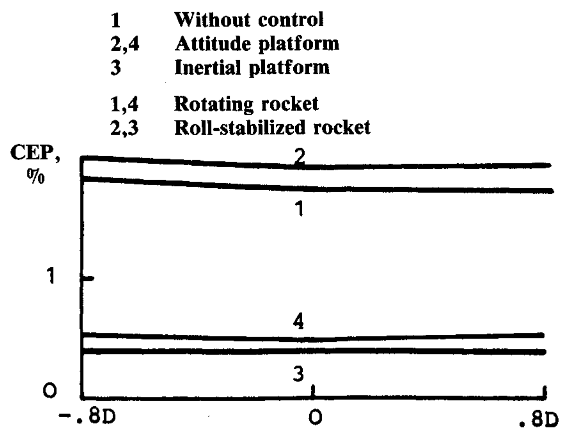

Fig. 13 Influence of static margin on CEP.

latter case. This would result in strict manufacturing tolerances.

It should be mentioned here that this CEP characteristic indicates relative changes and not absolute deviations. The latter are the lowest for the rocket with the inertial platform (by approximately 25% compared to the unguided rocket and 20% compared to the rocket with attitude control).

The influence of the stability on the CEP was investigated by varying the static margin within the range of -0.8 to $+0.8$ cal. with respect to the nominal value, which, on average, was approximately 1.3 cal. (the rocket was stable during the whole flight). Some of the results obtained are presented in Fig. 13. It was ascertained that this parameter exerts no influence on the CEP in the inertial platform case. The controller design was obviously very robust. However, decreasing static margin makes it easier for the control unit to compensate for the effect of the total-impulse tolerances since the demand for control deflections is decreasing.

Conclusions

An extensive aeromechanical investigation was undertaken for a rocket featuring a range of approximately 120 km and different control systems. The most important findings can be summarized as follows:

1) Utilization of attitude control does not offer any advantages compared to the ballistically flying, spinning rocket.

2) Combining attitude control with low rocket spinning would lead to a low CEP, provided that the total-impulse tolerances are small.

3) A control system utilizing an inertial platform (drift 1 deg/h) can easily achieve a CEP value of 0.4%. This system can cope with total-impulse tolerances of approximately $\pm 2\%$ by utilizing aerodynamic control only. For this purpose, a modulation of the prescribed values for the velocity vector angle γ^* with appropriate functions is necessary. Application of this control system would be the correct way to produce robust, low-cost rockets with the range indicated, since it can cope with high total-impulse tolerances without the need for thrust vector control.

4) The effect of slightly varying the axial-force coefficient is important when an inertial platform with the controller proposed here is used. Variations of $\pm 1\%$ result in a CEP increase of approximately 50%. However, improvements might be possible when designing a more sophisticated controller that considers the difference between actual and desired axial velocity. Small modifications to the normal-force coefficient did not cause any remarkable change to the CEP.

5) The influence of the static margin on the CEP is not very pronounced, since the designed rocket features high stability. However, a low static margin is preferable in order to compensate for the total-impulse tolerances in connection with an inertial platform (lower control deflection demand).

6) The controller designed here in connection with a strapdown inertial platform is not optimized. A more efficient controller should consider the differences between actual and desired rocket position, respectively axial velocity.

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